Tunneling escape in optical waveguide arrays with a boundary defect

Stefano Longhi
Dipartimento di Fisica and Istituto di Fotonica e Nanotecnologie del CNR, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milan, Italy

(Received 5 April 2006; published 10 August 2006)

Tuning of the decay law of light in an array of tunneling-coupled optical waveguides with a defect is theoretically demonstrated. An analytical form for the decay law in terms of a Neumann series is obtained in case of a semi-infinite array with a boundary defect waveguide. Different decay regimes, ranging from a near-exponential decay to an oscillatory or nonoscillatory power-law decay, are found in the parameter range where the boundary defect waveguide does not support localized modes.

DOI: 10.1103/PhysRevE.74.026602 PACS number(s): 42.25.−p, 42.79.−e

I. INTRODUCTION

Arrays of tunneling-coupled optical waveguides have received in the past recent years a continuous and increasing interest as experimentally accessible optical systems to test rather universal linear and nonlinear dynamical behavior of discrete systems [1,2]. Contrary to light propagation in bulk (homogeneous) media, in waveguide arrays the flow of light is ruled by energy transfer via the evanescent tails of respective guided modes and is influenced by the periodicity of the structure. In periodic arrays, tunneling-induced light coupling is responsible for remarkable deviations of discrete diffraction and refraction of beams from similar effects in bulk materials, as experimentally demonstrated in Refs. [3,4]. Propagation of discretized light can be further controlled in inhomogeneous waveguide arrays, where a localized or distributed perturbation is imposed on the otherwise periodic structure. For instance, for a distributed inhomogeneity in the form of a transverse index gradient the optical field performs photonic Bloch oscillations across the array [5–8], whereas for a periodic modulation of the waveguide axis dynamic localization, corresponding to diffraction cancellation, can be achieved [9,10]. The effect of a single defect or interface on linear and nonlinear propagation properties of the array has been also investigated [11–14]. In particular, in the linear propagation regime it has been theoretically shown and experimentally demonstrated that both an increase of the coupling constant and the variation of the effective index of a defect waveguide embedded in a periodic array can give rise to the formation of either staggered and unstaggered localized modes [13] and that a linear mode which is localized at the defect can escape at high power levels as phase matching with neighboring waveguides is achieved [11]. Recent works have been also devoted to study discrete light diffraction in a semi-infinite array of waveguides [16–18] and demonstrated both theoretically and experimentally the existence of discrete surface solitons in presence of a Kerr nonlinearity. An elegant method of images has been very recently proposed to determine the impulse response of a homogeneous semi-infinte array in the linear propagation regime [16]. In the presence of structural defects, most of previous studies of waveguide arrays or of semi-infinte arrays have been been concerned with the existence of linear or nonlinear localized modes at the defect site or with the scattering properties of linear or nonlinear waves. However, less attention has been devoted to study how the tunneling escape dynamics of light is influenced by the defect waveguide. Indeed, it is commonly believed that tunneling escape of light in an initially excited waveguide of the array shows an oscillatory power-law damped behavior, which is found both in infinite or semi-infinite arrays [16] and which is related to the characteristic impulse response of the array involving Bessel functions.

The aim of this paper is to investigate theoretically the basic features of tunneling escape dynamics in a semi-infinite array of tunneling-coupled optical waveguides with a boundary defect waveguide. A general procedure is introduced to provide a closed-form analytical solution to the problem of light diffraction in a semi-infinite array with a defect in the parameter range where no localized states are supported at the interface. The solution can be expressed either in the form of a Neumann series or as a contour integral in the complex plane. Different decay regimes, ranging from a near-exponential decay to an oscillatory or nonoscillatory power-law decay, are found and discussed.

The paper is organized as follows. In Sec. II the basic model of discrete light diffraction in a semi-infinite array of waveguides with a boundary defect is introduced, and the reflection and localization properties of the boundary are investigated. In Sec. III a general procedure to determine the discrete diffraction response of the array under single waveguide excitation is outlined, and closed-form analytical solutions of the problem are presented using either integral representations or Neumann series. In Sec. IV the tunneling escape dynamics from the boundary waveguide is studied in detail, and the existence of different decay regimes is predicted. In particular, as opposed to tunneling decay in homogeneous arrays, it is shown that a nearly exponential decay or a power-law monothonic decay (i.e., without oscillations) can be achieved under suitable design of the defect waveguide. Finally, in Sec. V the main conclusions are outlined.

II. REFLECTION AND LOCALIZATION IN A SEMI-INFINITE ARRAY WITH A BOUNDARY DEFECT

A. Basic model

The basic model considered in this work is represented by a semi-infinite array of weakly coupled single-mode optical

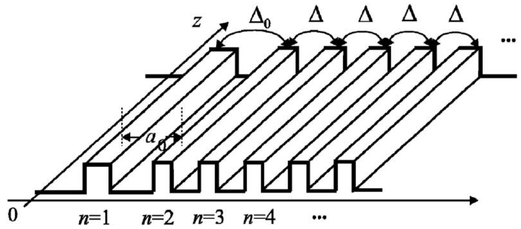

FIG. 1. Schematic of a semi-infinite array of tunneling-coupled optical waveguides with a defect boundary waveguide ($n=1$).

waveguides, as schematically depicted in Fig. 1. All the waveguides in the array are identical and equally spaced by the same distance a, except for the boundary waveguide $n=1$ which is assumed to be placed at a different distance a_0 from other waveguides and to have a different effective index of its guided mode. Because of weak coupling, light propagation in this system may be described by a set of coupled-mode equations in the tight-binding approximation for the modal amplitudes c_n in the nth waveguide of the chain (see, e.g., [13,16]):

$$i\dot{c}_n = -\Delta(c_{n+1}+c_{n-1}) \quad (n \geq 3), \tag{1a}$$

$$i\dot{c}_2 = -\Delta c_3 - \Delta_0 c_1, \tag{1b}$$

$$i\dot{c}_1 = -\Delta_0 c_2 - \sigma c_1, \tag{1c}$$

where the overdot denotes the derivatives with respect to the propagation distance z, Δ is the coupling constant between identical neighboring waveguides, Δ_0 is the coupling constant between the boundary waveguide ($n=1$) and its adjacent waveguide ($n=2$), and σ is the difference between the propagation constant of the boundary waveguide and the other waveguides of the array. In practice, the ratio $x=\Delta_0/\Delta$ and σ can be controlled by changing the ratio a_0/a and the width and/or index change of the boundary waveguide (see, e.g., [19]). Since we are interested in linear propagation properties of the array, nonlinear terms are not considered in Eqs. (1). We note that, in the absence of inhomogeneities—i.e., for $\Delta_0=\Delta$ and $\sigma=0$—the impulse response of the array under single waveguide excitation has been elegantly found in an analytical form by Makris and co-workers in recent works [15,16] using a method of images; our analysis extends the previous one when a boundary defect is included in the semi-infinite array.

B. Wave reflection and localization

Simple solutions to Eqs. (1) can be found in the form of either localized states at the boundary of the array or as a superposition of extended (propagating) Bloch waves incident and reflected from the boundary of the array. In the latter case we search for a solution of Eqs. (1) in the form

$$c_1 = A\exp[i\beta(Q)z], \tag{2a}$$

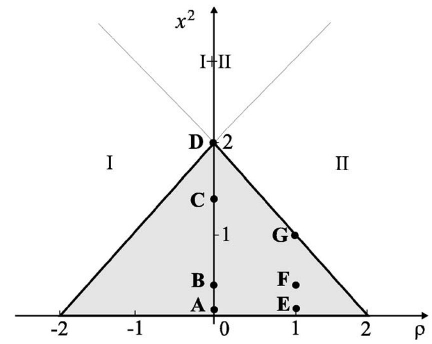

FIG. 2. Existence domain of bound modes of the semi-infinite array in the (ρ,x^2) plane. Regions I and II support one bound mode, corresponding to either one of the two values $\mu_{1,2}$ given by Eq. (6); region I+II supports both bound modes. In the triangular shaded region no bound modes are supported by the array. Points $A-G$ correspond to numerical values used in the simulations shown in Figs. 4–8.

$$c_n = \{\exp[-iQ(n-2)] + r(Q)\exp[iQ(n-2)]\} \times \exp[i\beta(Q)z] \quad (n \geq 2), \tag{2b}$$

where $-\pi < Q \leq \pi$ is the wave number of the Bloch waves, $\beta(Q)=2\Delta\cos(Q)$ is the dispersion curve of the homogeneous part of the array, $r(Q)$ is the reflection coefficient, and A is the mode amplitude at the boundary waveguide $n=1$. Substitution of Eqs. (2) into Eqs. (1), after simple algebra one can derive the following expression for the reflection coefficient $r(Q)$:

$$r(Q) = -\frac{x^2 - \exp(iQ)(2\cos Q - \rho)}{x^2 - \exp(-iQ)(2\cos Q - \rho)}, \tag{3}$$

where we have set $x \equiv \Delta_0/\Delta$ and $\rho \equiv \sigma/\Delta$. Furthermore, the mode amplitude A at the defect waveguide is given by

$$A = \frac{\Delta_0(1+r)}{\beta - \sigma} = \frac{2ix\sin Q}{x^2 - \exp(-iQ)(2\cos Q - \rho)}. \tag{4}$$

Note that, as expected, $|r|=1$, which means that propagating waves incident onto the boundary are completely reflected.

The waveguide defect at the boundary of the array can also support localized modes. We can search for localized modes at the boundary in the form

$$c_1 = A\exp(i\beta), \tag{5a}$$

$$c_n = B\mu^{-(n-2)}\exp(i\beta) \quad (n \geq 2), \tag{5b}$$

where $\beta=\Delta(\mu+1/\mu)$, with the condition $|\mu|>1$ for localization. Substitution of the ansatz (5) into Eqs. (1) yields for μ one of the two possible values:

$$\mu_{1,2} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 + x^2 - 1}. \tag{6}$$

The domain of existence of bound modes in the plane (ρ,x^2) is depicted in Fig. 2. Note that such a domain resembles the one previously found for an infinite array with a defect waveguide [13] and that in the triangular region defined by the inequality $|\rho|<2-x^2$ no bound modes exist. Note also that the homogeneous semi-infinite array, corresponding to $\rho=0$ and $x=1$, falls inside such a triangular region. Since we are interested in the study of tunneling escape of light from the boundary defect waveguide, in the following analysis we will limit our attention to parameter values where the semi-infinite array does not support any bound mode.

III. IMPULSE RESPONSE OF THE SEMI-INFINITE ARRAY

In this section we present a general procedure to construct the impulse response of the array—i.e., the solution to the coupled-mode equations (1) corresponding to a single waveguide excitation at the input plane, $c_n(0)=\delta_{n,\bar{n}}$, with $\bar{n}\geq 1$. In particular, since we are interested in determining the tunneling decay law of the boundary defect waveguide due to its coupling with the other waveguides in the array, specific results will be presented for the case $\bar{n}=1$. In the parameter range where no bound modes exist, the most general solution to Eqs. (1) can be constructed as a superposition of Bloch waves (2) with an arbitrary spectral amplitude $F(Q)$; i.e., one can write

$$c_1(z)=\int_{-\pi}^{\pi} dQF(Q)\frac{2ix\sin Q}{x^2-\exp(-iQ)(2\cos Q-\rho)}\times\exp[2i\Delta z\cos Q], \tag{7a}$$

$$c_n(z)=\int_{-\pi}^{\pi} dQF(Q)\left\{\exp[-iQ(n-2)]-\frac{x^2-\exp(iQ)(2\cos Q-\rho)}{x^2-\exp(-iQ)(2\cos Q-\rho)}\exp[iQ(n-2)]\right\}\exp[2i\Delta z\cos Q]\quad(n\geq 2). \tag{7b}$$

Let us first assume a spectrum of the form

$$F^{(N)}(Q)=\frac{x^2-\exp(-iQ)(2\cos Q-\rho)}{2\pi}\exp(-iNQ), \tag{8}$$

where N is an integer number. In this case, the integrals in Eqs. (7) can be analytically calculated, yielding the elementary solutions

$$c_1^{(N)}(z)=xJ_{N-1}(2\Delta z)\exp[i\pi(N-1)/2]-xJ_{N+1}(2\Delta z)\exp[i\pi(N+1)/2], \tag{9a}$$

$$\begin{aligned}c_n^{(N)}(z)=&(x^2-1)J_{n-2+N}(2\Delta z)\exp[i\pi(n-2+N)/2]\\&-(x^2-1)J_{N-n+2}(2\Delta z)\exp[i\pi(N-n+2)/2]\\&-J_{N+n}(2\Delta z)\exp[i\pi(N+n)/2]+J_{N-n}(2\Delta z)\exp[i\pi(N-n)/2]+\rho J_{N+n-1}(2\Delta z)\exp[i\pi(N+n-1)/2]\\&-\rho J_{N-n+1}(2\Delta z)\exp[i\pi(N-n+1)/2]\quad(n\geq 2),\end{aligned} \tag{9b}$$

where J_n is the Bessel function of first kind and of order n. We can then search for a solution of the coupled-mode equations (1) satisfying the initial condition $c_n(0)=\delta_{n,\bar{n}}$ as a superposition of elementary solutions (9); i.e., we set

$$c_n(z)=\sum_{N=0}^{+\infty}T_N c_n^{(N)}(z), \tag{10}$$

where the coefficients T_N have to be determined from the desired initial condition. Note that the series (10) corresponds to an expansion of $c_n(z)$ on the basis of Bessel functions and it thus represents the Neumann series expansion of the searched solution. Taking into account that $J_n(0)=\delta_{n,0}$ and assuming $T_0=0$ [20], from Eqs. (9) and (10) one has

$$c_1(0)=xT_1, \tag{11a}$$

$$c_n(0)=T_n-\rho T_{n-1}-(x^2-1)T_{n-2}\quad(n\geq 2). \tag{11b}$$

In order to determine the coefficients T_n, we should distinguish two cases.

(i) *Excitation of the boundary defect waveguide*. In order to get $c_n(0)=\delta_{n,1}$, from Eqs. (11) it follows that the coefficients T_n have to satisfy the recurrence relation

$$T_n-\rho T_{n-1}-(x^2-1)T_{n-2}=0, \tag{12}$$

with the initial conditions $T_0=0$ and $T_1=1/x$. The corresponding solution reads explicitly

$$T_n=\frac{1}{x(\mu_1-\mu_2)}(\mu_1^n-\mu_2^n), \tag{13}$$

where μ_1 and μ_2 are given by Eq. (6). It should be noted that, since we consider the parameter region where $|\mu_{1,2}|<1$, T_n goes to zero as $n\to\infty$ and the Neumann series (10) is convergent. Note that the spectrum $F(Q)$ corresponding to this solution is given by

$$F(Q)=\sum_{N=0}^{\infty}T_N F^{(N)}(Q)=\frac{1}{2\pi x}\times\frac{[x^2-\exp(-iQ)(2\cos Q-\rho)]\exp(-iQ)}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]}. \tag{14}$$

From Eqs. (7) and (14), the following integral representation for the impulse response $c_n(z)$ of the array, corresponding to the excitation of the boundary waveguide $\bar{n}=1$, can be written:

$$c_1(z)=\frac{1}{2\pi}\int_{-\pi}^{\pi} dQ\frac{1-\exp(-2iQ)}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]}\exp(2i\Delta z\cos Q), \tag{15a}$$

$$c_n(z) = \frac{i}{\pi x}\int_{-\pi}^{\pi} dQ \exp(-iQ)\frac{(2\cos Q-\rho)\sin[(n-1)Q]-x^2\sin[(n-2)Q]}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]}\exp(2i\Delta z\cos Q) \quad (n\geqslant 2). \tag{15b}$$

It should be noted that, in the general case, the Neumann series (10) contains an infinite number of terms. There are, however, a few special cases in which the series has a finite number of terms and the impulse response of the semi-infinite array can be expressed in a simple form. The first case is that of a homogeneous semi-infinite array, which was previously considered in Ref. [16]. In this case, corresponding to $x=1$ and $\rho=0$, one has $\mu_1=\mu_2=0$ and in the Neumann expansion (10) only the coefficient T_1 is nonvanishing and equal to $T_1=1/x=1$. One then obtains

$$c_n(z) = i^{(n-1)}[J_{n+1}(2\Delta z)+J_{n-1}(2\Delta z)] \quad (n\geqslant 1). \tag{16}$$

A second special case in which the Neumann series (10) has a finite number of elements corresponds to the limiting case $x=1$ and $\rho=\pm 1$. Note that this case corresponds to a boundary point in the diagram of Fig. 2, for which $\mu_1=\pm 1$ and $\mu_2=0$. For $\rho=1$, directly from Eqs. (15) one readily obtains

$$c_n(z) = i^n[J_n(2\Delta z)-iJ_{n-1}(2\Delta z)] \quad (n\geqslant 1), \tag{17}$$

whereas for $\rho=-1$ one has

$$c_n(z) = -i^n[J_n(2\Delta z)+iJ_{n-1}(2\Delta z)] \quad (n\geqslant 1). \tag{18}$$

(ii) *Excitation of an internal waveguide*. Suppose now that a waveguide internal to the array is initially excited—i.e., that $c_n(0)=\delta_{n,\bar{n}}$ with $\bar{n}\geqslant 2$. Following a procedure similar to that described in the previous case, from Eqs. (11) one can show that the initial condition is now satisfied by assuming $T_n=0$ if $n\leqslant\bar{n}-1$ and

$$T_n = \frac{1}{\mu_1-\mu_2}(\mu_1^{n-\bar{n}+1}-\mu_2^{n-\bar{n}+1}) \tag{19}$$

for $n\geqslant\bar{n}$. The corresponding spectrum, as obtained from Eqs. (8) and (19), then reads

$$F(Q) = \frac{1}{2\pi}\frac{[x^2-\exp(-iQ)(2\cos Q-\rho)]\exp(-iQ\bar{n})}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]}. \tag{20}$$

Finally, an integral representation of the Neumann series (10) similar to Eqs. (15) can be written:

$$c_1(z) = \frac{x}{2\pi}\int_{-\pi}^{\pi} dQ\frac{\exp[-iQ(\bar{n}-1)]-\exp[-iQ(\bar{n}+1)]}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]} \times\exp(2i\Delta z\cos Q), \tag{21a}$$

$$c_n(z) = \frac{i}{\pi}\int_{-\pi}^{\pi} dQ\exp(-iQ\bar{n}) \times\frac{(2\cos Q-\rho)\sin[(n-1)Q]-x^2\sin[(n-2)Q]}{[1-\mu_1\exp(-iQ)][1-\mu_2\exp(-iQ)]} \times\exp(2i\Delta z\cos Q) \quad (n\geqslant 2). \tag{21b}$$

As in the previous case (i), simple analytical expressions for $c_n(z)$, corresponding to a finite number of terms in the Neumann expansion (10), can be given in two special cases. The first case corresponds to the semi-infinite homogeneous array $x=1$ and $\rho=0$, which yields

$$c_n(z) = i^{n-\bar{n}}J_{n-\bar{n}}(2\Delta z)-i^{n+\bar{n}}J_{n+\bar{n}}(2\Delta z) \quad (n\geqslant 1). \tag{22}$$

The other case corresponds to $x=1$ and $\rho=\pm 1$. For $\rho=1$ one has

$$c_n(z) = i^{n-\bar{n}}J_{n-\bar{n}}(2\Delta z)+i^{n+\bar{n}-1}J_{n+\bar{n}-1}(2\Delta z) \quad (n\geqslant 1), \tag{23}$$

whereas for $\rho=-1$ one has

$$c_n(z) = i^{n-\bar{n}}J_{n-\bar{n}}(2\Delta z)-i^{n+\bar{n}-1}J_{n+\bar{n}-1}(2\Delta z) \quad (n\geqslant 1). \tag{24}$$

IV. TUNNELING ESCAPE DYNAMICS

In this section we study in detail the tunneling escape dynamics $c_1=c_1(z)$ describing the decay law of light trapped at the boundary waveguide of the array when it is excited at the input plane—i.e., when $c_n(0)=\delta_{n,1}$. The main result of this analysis is that different qualitative behavior of the decay can be attained by tuning the defect parameters ρ and x.

A. The $\rho=0$ case

Let us first consider the case in which all the waveguides in the array are identical, but the distance a_0 of the boundary waveguide is in general different than the periodicity a of the semi-infinite array. In this case, one has $\rho=0$ and $x\neq 1$; in fact, if w_0 is the characteristic size of the individual waveguide mode, one has $x\simeq\exp[-(a_0-a)/w_0]$ [19], resulting in $x<1$ if $a_0>a$ and $x>1$ in the opposite case. The decay law $c_1(z)$ of mode amplitude trapped in the boundary waveguide $n=1$ can be expressed by a Neumann series [Eq. (10)] which has the integral representation given by Eq. (15a). Taking into account the identity $J_{n+1}(\xi)+J_{n-1}(\xi)=(2n/\xi)J_n(\xi)$, from Eqs. (9a), (10), and (13) the Neumann series takes the explicit form

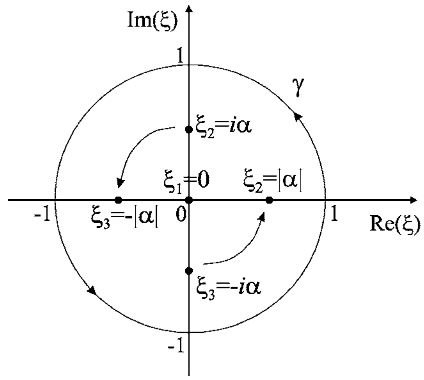

FIG. 3. Schematic of the contour γ in the complex ξ plane entering in Eq. (26). Points $\xi_{1,2,3}$ correspond to the poles internal to the contour γ. For $x<1$ the poles ξ_2 and ξ_3 lie on the imaginary axis, whereas for $1<x<\sqrt{2}$ they are located on the real axis.

$$c_1(z) = \frac{1}{\Delta z}\sum_{N=0}^{\infty} \alpha^{2N}(2N+1)J_{2N+1}(2\Delta z), \qquad (25)$$

where we have set $\alpha \equiv (1-x^2)^{1/2}$. It is also worth noting that, after setting $\xi = \exp(iQ)$, the integral representation (15a) can be written as a contour integral in the complex ξ plane

$$c_1(z) = \frac{1}{2\pi i}\oint_\gamma d\xi \frac{\xi^2-1}{\xi(\xi^2+\alpha^2)} \exp\left[i\Delta z\left(\xi + \frac{1}{\xi}\right)\right], \qquad (26)$$

where the contour γ is the unit circle $|\xi|=1$ (see Fig. 3). The integral (26) can be evaluated by use of the residue theorem. Note that there are three poles inside the contour γ, one located at $\xi=\xi_1=0$ and the other two located on the imaginary axis at $\xi=\xi_{2,3}=\pm i\alpha$ for $x<1$ or on the real axis at $\xi=\xi_{2,3}=\pm|\alpha|$ for $1<x<\sqrt{2}$. At $x=1$ (homogeneous semi-infinite array), there is only one singularity at $\xi=0$, and from the residue theorem one recovers the simple result derived in the previous section [Eq. (16) with $n=1$]:

$$c_1(z) = \frac{1}{\Delta z} J_1(2\Delta z). \qquad (27)$$

For $x<1$, corresponding to a weak coupling of the boundary waveguide with the array, the residue associated with the singularity $\xi_3=-i\alpha$ yields an exponentially decaying term, whereas the sum of residues at $\xi_1=0$ and $\xi_2=i\alpha$ yields a bounded function $s(z)$, which can be written as a Neumann series. Precisely, one can formally write

$$c_1(z) = \sqrt{\mathcal{Z}}\exp(-\gamma_0 z/2) + s(z), \qquad (28)$$

where $\sqrt{\mathcal{Z}} = (1+\alpha^2)/(2\alpha^2)$,

$$\gamma_0 = 2\Delta\frac{1-\alpha^2}{\alpha} = 2\Delta\frac{x^2}{\sqrt{1-x^2}} \qquad (29)$$

is the decay rate of the exponential term, and

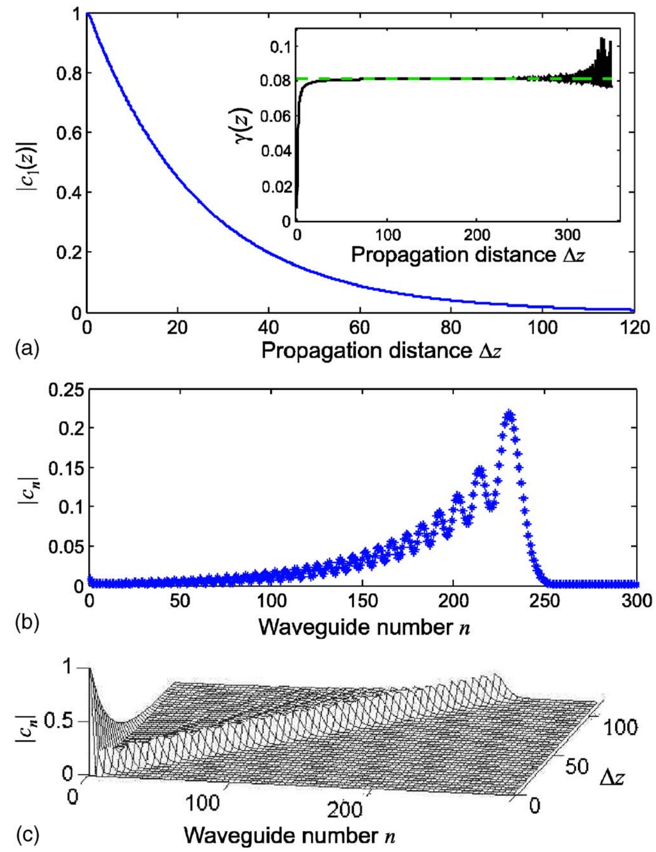

FIG. 4. (Color online) (a) Behavior of mode amplitude $|c_1(z)|$ versus propagation distance for $x=0.2$ and $\rho=0$. The inset shows the behavior of the local decay rate $\gamma(z)$ (solid curve) defined by Eq. (32). The dashed horizontal curve in inset is the theoretical value of the decay rate γ_0 as given by Eq. (29). (b) Distribution of mode amplitudes $|c_n|$ of various waveguides at the final propagation distance ($\Delta z=120$) of (a). (c) Three-dimensional plot showing the evolution of mode amplitudes $|c_n(z)|$ versus propagation distance.

$$s(z) = J_0(2\Delta z) + \left(1+\frac{1}{\alpha^2}\right)\left[\frac{1}{2}\sum_{l=-\infty}^{\infty}\frac{J_l(2\Delta z)}{\alpha^l} - \sum_{l=0}^{\infty}\frac{J_{2l}(2\Delta z)}{\alpha^{2l}}\right] \qquad (30)$$

is the correction to the exponential decay term. The decomposition (28) is meaningful in the weak coupling regime ($\Delta_0 \ll \Delta$—i.e., $x\to 0$) since, in this limit, one can show that the nonexponential contribution $s(z)$ is small and of the order $\sim O(x^2)$. In fact, an asymptotic expansion of Eq. (30) with respect to x yields

$$s(z) = -\frac{x^2}{2}\left[J_0(2\Delta z) + \sum_{N=0}^{\infty} J_{2N+1}(2\Delta z) - \frac{J_1(2\Delta z)}{2} - \frac{1}{2}\right] + O(x^4). \qquad (31)$$

As an example, Figs. 4–6 show the decay behavior of $c_1(z)$ for a few values of x. In the figures, three-dimensional plots showing the full light escape dynamics to the adjacent waveguides are also shown. For small values of x, an almost exponential decay is observed (Fig. 4, corresponding to point

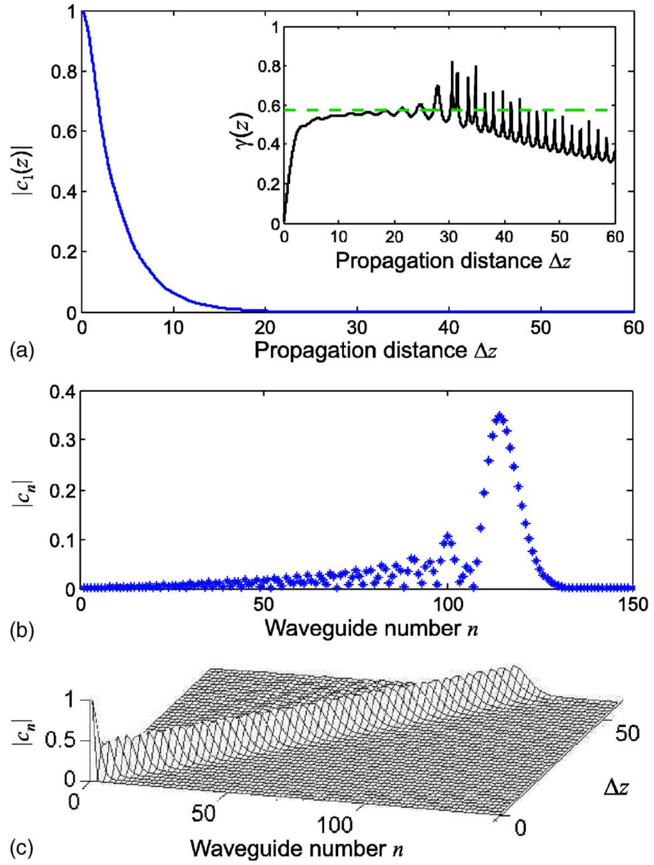

FIG. 5. (Color online) Same as Fig. 4, but for $x=0.5$ and $\rho=0$.

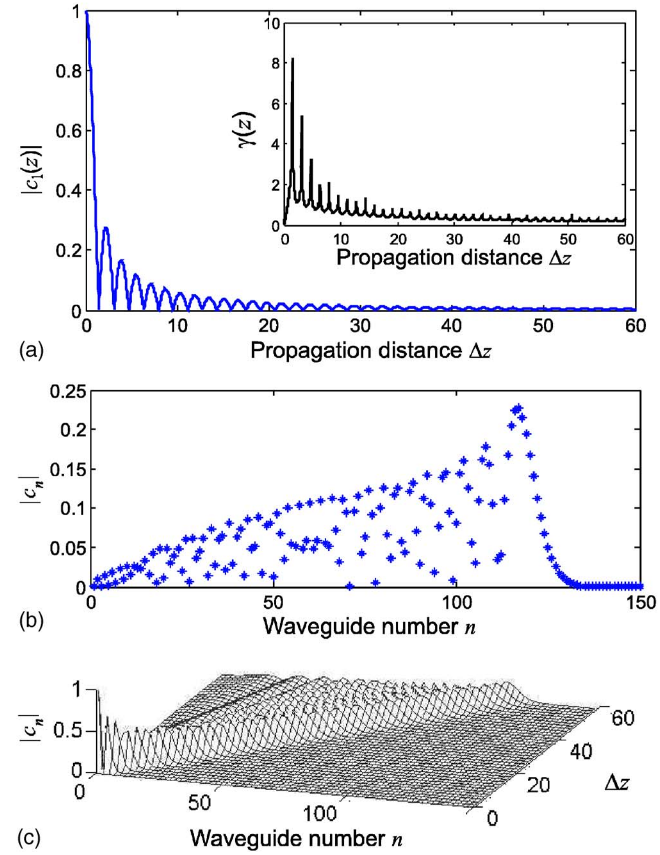

FIG. 6. (Color online) Same as Fig. 4, but for $x=1.2$ and $\rho=0$.

A in Fig. 2). Deviations from the exponential decay are at best visualized by introducing a "local" decay rate $\gamma(z)$ defined by

$$\gamma(z) = -\frac{1}{z}\ln|c_1(z)|^2. \tag{32}$$

Any deviation of $\gamma(z)$ from γ_0 is a signature of nonexponential decay. An inspection of Fig. 4 clearly shows that, for small values of x, the decay is to a good approximation exponential with a decay rate γ_0 [first term in Eq. (28)], however, deviations from the exponential decay are visible both at $z\rightarrow 0$ and at $z\rightarrow\infty$. In fact, a perturbative analysis of Eqs. (1) close to $z=0$ clearly reveals that the decay law close to $z=0$ is always parabolic [i.e., $c_1(z)\sim 1-(\Delta_0^2/2)z^2$ as $z\rightarrow 0$], whereas for $z\rightarrow\infty$ the decay is described by an oscillating power-law relation. The asymptotic behavior of $c_1(z)$ for $z\rightarrow\infty$ can be determined from the integral representation (15a) by means of the method of the stationary phase and reads explicitly (see the Appendix for technical details)

$$c_1(z) \sim \frac{1}{\sqrt{\pi}}\frac{x^2}{(2-x^2)^2}\frac{1}{(\Delta z)^{3/2}}\cos\left(2\Delta z-\frac{3\pi}{4}\right) \quad \text{as } z\rightarrow\infty. \tag{33}$$

To summarize, for weak coupling (small values of x) the decay is exponential with small deviations solely at short and long propagation distances z. It should be noted that such deviations are a rather universal phenomenon generally encountered in problems of the quantum-mechanical decay of metastable states [21] or in problems of the tunneling escape of a particle from a potential barrier [22,23]. In particular, the parabolic law for the decay in the early stage can be exploited for an optical realization of the quantum Zeno effect using a semi-infinite array of tunneling-coupled optical waveguides [24]. As x is increased toward 1, the decay appreciably deviates from an exponential law (see Fig. 5, corresponding to point B in Fig. 2) at any propagation distance, and this behavior is strongly enhanced as x gets larger than 1 (see Fig. 6, corresponding to point C in Fig. 2), at which the poles ξ_2 and ξ_3 in the complex integral of Eq. (26) move from the imaginary to the real ξ axis passing through zero (see Fig. 3) and the first term in Eq. (28) changes from exponential to oscillatory (γ_0 complex). Note that the asymptotic decay (33), strictly valid for $x<\sqrt{2}$, indicates that the light escape from the boundary waveguide via tunneling is *higher* than the usual power-law decay $\sim 1/\sqrt{z}$ of homogeneous infinte arrays; this circumstance was recently noted in Ref. [16] for the special case of a homogeneous semi-infinite array ($x=1$). However, it is a rather general property. As x is increased to reach the boundary $x=\sqrt{2}$ (point D in Fig. 2), a divergence occurs in Eq. (33) and the function $c_1(z)$ can be directly obtained from Eq. (15a) after observing that $\mu_1=1$ and $\mu_2=-1$. In this case, one simply obtains

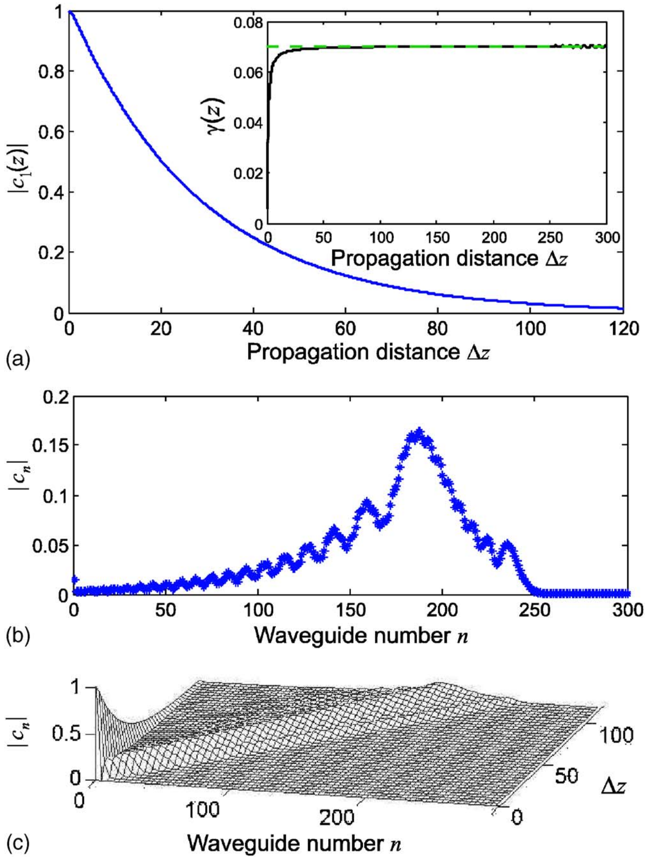

FIG. 7. (Color online) Same as Fig. 4, but for $x=0.2$ and $\rho=1$.

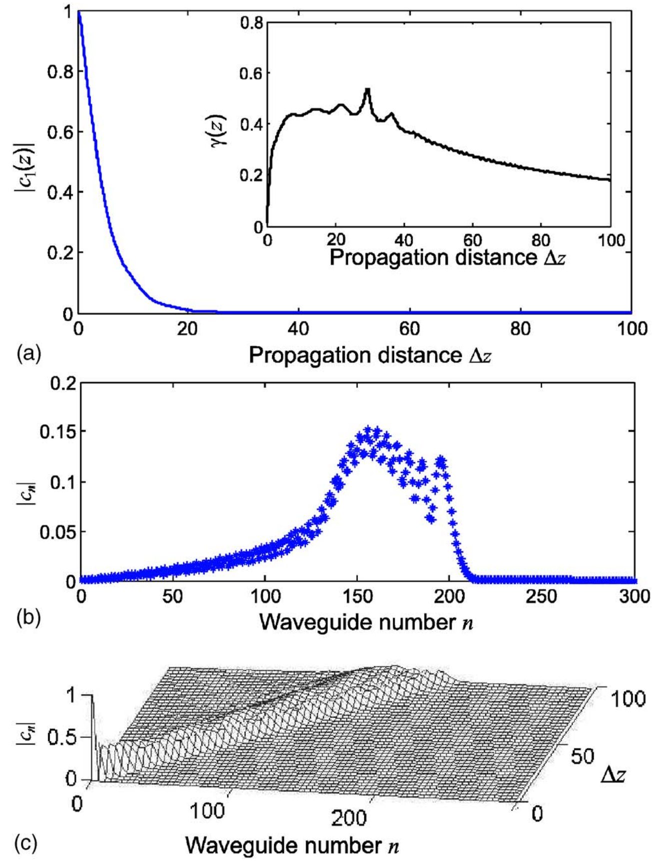

FIG. 8. (Color online) Same as Fig. 4, but for $x=0.5$ and $\rho=1$.

$$c_1(z) = J_0(2\Delta z), \tag{34}$$

indicating that the decay law is the same as that of an infinite homogeneous array (power law decay $\sim 1/\sqrt{z}$ with oscillatory behavior). As shown in the Appendix, inside the triangular domain of Fig. 2 the asymptotic decay is governed by a power law $\sim 1/z^{3/2}$; however, at the lateral boundaries of the triangle the decay is governed by a power law $\sim 1/z^{1/2}$. This lowering of the power exponent in the decay law is a signature of the transition from a region of decay to a region where bound states do exist.

B. The $\rho \neq 0$ case

In case where the boundary waveguide shows a different effective index from the adjacent waveguides in the array, in addition to $x \neq 1$ one also has $\rho \neq 0$ in the coupled-mode equations (1). For a given value of $\rho \neq 0$, by varying the coupling x, the qualitative behavior of tunneling decay dynamics is not substantially modified from the $\rho=0$ case. As shown in Fig. 7, for, e.g., a small coupling corresponding to point E in Fig. 2, a nearly exponential decay is still found, apart for deviations in the first and last propagation distances. To calculate the decay rate γ_0 in this case, note that Eq. (26) is now replaced by the following one:

$$c_1(z) = \frac{1}{2\pi i}\oint_\gamma d\xi \frac{\xi^2 - 1}{\xi(\xi-\mu_1)(\xi-\mu_2)} \exp\left[i\Delta z\left(\xi + \frac{1}{\xi}\right)\right]. \tag{35}$$

A decomposition similar to Eq. (28) can be then stated, the exponential decaying term arising from the residue $\xi=\mu$, with $\mu=\mu_1$ if $\mathrm{Im}(\mu_1+1/\mu_1)>0$ or $\mu=\mu_2$ if $\mathrm{Im}(\mu_2+1/\mu_2)>0$. In this case, the decay rate γ_0 is given by

$$\gamma_0 = 2\Delta\, \mathrm{Im}\left(\mu + \frac{1}{\mu}\right). \tag{36}$$

As the coupling x is increased, deviations from the exponential decay are visible at any propagation distance with an asymptotic power-law decay; however, as opposed to the $\rho=0$ case, the power-law decay is now monotonic, and not oscillatory, as shown in Fig. 8, corresponding to point F in Fig. 2. As x is further increased to reach the boundary of the triangular domain of Fig. 2 (point G), the power-law decay changes from $\sim 1/\sqrt{z}$ to $\sim 1/z^{3/2}$, according to the asymptotic analysis given in the Appendix. In Figs. 7 and 8 we have shown tunneling decay results corresponding to $\rho>0$; however a similar behavior is found for $\rho<0$. Therefore the main feature of the $\rho \neq 0$ case is to avoid the occurrence of oscillations in the decay process.

V. CONCLUSIONS

In this work we have analytically studied the tunneling-induced decay dynamics of light in a semi-infinite array of coupled optical waveguides with a boundary defect waveguide. An analytical form of the impulse response of the array has been presented in terms of a Neumann series, and different decay regimes, ranging from a near-exponential decay to an oscillatory or nonoscillatory power-law decay, have been found in the parameter range where the boundary defect waveguide does not support localized modes. At the boundary of existence of bound modes, it has been shown that a lowering of the exponent of the power decay law is achieved. It is envisaged that the different decay regimes predicted by the present analysis may be experimentally observed in semi-infinite arrays of tunneling-coupled optical waveguides using near-field scanning optical microscopy techniques to quantitative measure the power flow of light along the boundary waveguide of the array [25].

APPENDIX: ASYMPTOTIC BEHAVIOR OF THE DECAY LAW

In this appendix the asymptotic behavior for $z \to \infty$ of the decay law $c_1(z)$ is derived starting from the integral representation of $c_1(z)$, which can be written in the form [see Eq. (15a)]

$$c_1(z) = \frac{1}{2\pi}\int_{-\pi/2}^{3\pi/2} dQ R(Q)\exp[iz\beta(Q)], \tag{A1}$$

where we have set $\beta(Q) \equiv 2\Delta \cos Q$ and $R(Q) \equiv 2i \exp(-iQ)\sin Q/\{[1-\mu_1 \exp(-iQ)][\mu_2-\exp(-iQ)]\}$ and where we have assumed $[-\pi/2, 3\pi/2]$ as the integration interval. The asymptotic behavior of $c_1(z)$ as $z \to \infty$ can be analytically determined by means of the method of the stationary phase. To this aim, let us note that there are two points in the integration interval where the derivative $d\beta/dQ$ vanishes—namely, $Q=0$ and $Q=\pi$; therefore, two main contributions to the integral have to be considered. However, at these two stationary points the function $R(Q)$ usually vanishes. Indeed, one can easily prove that $R(0)=R(\pi)=0$ for (ρ, x^2) strictly internal to the triangular domain of Fig. 2, whereas $R(Q)$ is nonvanishing at either $Q=0$ or $Q=\pi$ at the lateral boundaries $x^2 = \pm\rho+2$ of the triangle in Fig. 2. We have therefore to distinguish two cases.

(i) (ρ, x^2) *internal to the triangle of Fig. 2*. In this case, before applying the method of the stationary phase one has to perform an integration by part in Eq. (A1). This yields

$$c_1(z) = \frac{1}{2\pi\Delta z}\int_{-\pi/2}^{3\pi/2} dQ G(Q)\exp[iz\beta(Q)], \tag{A2}$$

where we have set

$$G(Q) \equiv \frac{d}{dQ}\frac{\exp(-iQ)}{[1-\mu_1 \exp(-iQ)][1-\mu_2 \exp(-iQ)]} = \frac{-i\exp(-iQ)[1-\mu_1\mu_2 \exp(-2iQ)]}{[1-\mu_1 \exp(-iQ)]^2[1-\mu_2 \exp(-iQ)]^2}. \tag{A3}$$

A direct application of the method of the stationary phase to Eq. (A2) is now possible, yielding

$$c_1(z) \sim \frac{1}{2\sqrt{\pi}}\frac{1}{(\Delta z)^{3/2}}\left[\frac{(1-\mu_1\mu_2)\exp(2i\Delta z - i3\pi/4)}{(1-\mu_1)^2(1-\mu_2)^2} + \frac{(1-\mu_1\mu_2)\exp(-2i\Delta z + i3\pi/4)}{(1+\mu_1)^2(1+\mu_2)^2}\right]. \tag{A4}$$

As a special case, note that for $\rho=0$ one has $\mu_1 = -\mu_2 = (x^2-1)^{1/2}$; from Eq. (A4), one then obtains

$$c_1(z) \sim \frac{1}{\sqrt{\pi}(\Delta z)^{3/2}}\frac{1+\mu_1^2}{(1-\mu_1^2)^2}\cos(2\Delta z - 3\pi/4), \tag{A5}$$

which indicates that the decay is oscillatory. For $\rho \neq 0$, one of the two exponential terms in Eq. (A4) is prevalent, corresponding to a partial suppression of the oscillatory behavior. As the boundary of the triangle in Fig. 2 is approached, one of the two terms in Eq. (A4) strongly prevails over the other one and the decay is monotonic (without oscillations).

(ii) (ρ, x^2) *at the boundary of the triangle of Fig. 2*. In this case the function $R(Q)$ does not vanish at either $Q=0$ or $Q=\pi$, indicating that the decay is now ruled by the $\sim 1/\sqrt{z}$ decay law rather than $\sim 1/z^{3/2}$. Specifically, at the boundary $x^2 = -\rho+2$, one has $\mu_1 = \rho-1$ and $\mu_2 = 1$, so that the application of the method of the stationary phase to Eq. (A1) yields

$$c_1(z) = \sqrt{\frac{\pi}{\Delta z}}\frac{2}{1-\mu_1}\exp(2i\Delta z - i\pi/4). \tag{A6}$$

At the boundary $x^2 = \rho+2$, one has $\mu_1 = \rho+1$ and $\mu_2 = -1$, so that the application of the method of the stationary phase to Eq. (A1) now yields

$$c_1(z) = \sqrt{\frac{\pi}{\Delta z}}\frac{2}{1+\mu_1}\exp(-2i\Delta z + i\pi/4). \tag{A7}$$

We note that, besides showing a decay law with a lower power exponent, at the boundary of the triangle of Fig. 2 the asymptotic decay is nonoscillatory. An exception occurs for $\rho=0$ and $x^2=1$—i.e., at the upper vertex of the triangle of Fig. 2. In this case from Eq. (A1) one obtains the exact result $c_1(z) = J_0(2\Delta z)$, which shows an oscillatory decay with the same power exponent.

[1] F. Lederer and Y. Silberberg, Opt. Photonics News **53**, 48 (2002).
[2] D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature (London) **424**, 817 (2003).
[3] S. Somekh, E. Garmire, A. Yariv, H. L. Garvin, and R. G. Hunsperger, Appl. Phys. Lett. **22**, 46 (1973).
[4] T. Pertsch, T. Zentgraf, U. Peschel, A. Bräuer, and F. Lederer, Phys. Rev. Lett. **88**, 093901 (2002).
[5] R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83**, 4756 (1999).
[6] T. Pertsch, P. Dannberg, W. Elflein, A. Bräuer, and F. Lederer, Phys. Rev. Lett. **83**, 4752 (1999).
[7] G. Lenz, I. Talanina, and C. M. de Sterke, Phys. Rev. Lett. **83**, 963 (1999).
[8] H. Trompeter, T. Pertsch, F. Lederer, D. Michaelis, U. Streppel, A. Bräuer, and U. Peschel, Phys. Rev. Lett. **96**, 023901 (2006); Henrike Trompeter, Wieslaw Krolikowski, Dragomir N. Neshev, Anton S. Desyatnikov, Andrey A. Sukhorukov, Yuri S. Kivshar, Thomas Pertsch, Ulf Peschel, and Falk Lederer, *ibid.* **96**, 053903 (2006).
[9] G. Lenz, R. Parker, M. C. Wanke, and C. M. de Sterke, Opt. Commun. **218**, 87 (2003).
[10] S. Longhi, Opt. Lett. **30**, 2137 (2005).
[11] U. Peschel, R. Morandotti, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Appl. Phys. Lett. **75**, 1384 (1999).
[12] R. Morandotti, H. S. Eisenberg, D. Mandelik, Y. Silberberg, D. Modotto, M. Sorel, C. R. Stanley, and J. S. Aitchison, Opt. Lett. **28**, 834 (2003).
[13] H. Trompeter, U. Peschel, T. Pertsch, F. Lederer, U. Streppel, D. Michaelis, and A. Bräuer, Opt. Express **11**, 3404 (2003).
[14] L. Morales-Molina and R. A. Vicencio, Opt. Lett. **31**, 966 (2006).
[15] Konstantinos G. Makris, Sergiy Suntsov, Demetrios N. Christodoulides, George I. Stegeman, and Alain Hache, Opt. Lett. **30**, 2466 (2005).
[16] K.G. Makris and D.N. Christodoulides, Phys. Rev. E **73**, 036616 (2006).
[17] S. Suntsov, K.G. Makris, D.N. Christodoulides, G.I. Stegeman, A. Hache, R. Morandotti, H. Yang, G. Salamo, and M. Sorel, Phys. Rev. Lett. **96**, 063901 (2006).
[18] Mario Molina, Rodrigo Vicencio, and Yuri Kivshar, Opt. Lett. **31**, 1693 (2006).
[19] M. J. Ablowitz and Z. H. Musslimani, Physica D **184**, 276 (2003).
[20] The values of $c_n(z)$ turn out to be independent of the coefficient T_0, which is thus set to zero for convenience. In fact, from Eqs. (9) one has $c_n^{(0)}(z) \equiv 0$.
[21] L. Fonda, G. C. Ghirardi, and A. Rimini, Rep. Prog. Phys. **41**, 587 (1978).
[22] M. Razavy, *Quantum Theory of Tunneling* (World Scientific, River Edge, NJ, 2003).
[23] W. van Dijk and Y. Nogami, Phys. Rev. Lett. **83**, 2867 (1999); T. Koide and F. M. Toyama, Phys. Rev. A **66**, 064102 (2002); S. De Leo and P. P. Rotelli, *ibid.* **70**, 022101 (2004).
[24] S. Longhi (unpublished).
[25] A. L. Campillo, J. W. P. Hsu, K. R. Parameswaran, and M. M. Fejer, Opt. Lett. **28**, 399 (2003).